Differenciation

$u^n = n u^{n-1} u'$

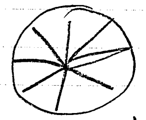
① □ $l = l_0 (1 + \lambda T)$

$\frac{dl}{dT} = l_0 \lambda \iff \boxed{\begin{array}{l} dl = l_0 \lambda dT \\ \Delta l = l_0 \lambda \Delta t \end{array}}$

$l = l_0 (1 + \lambda T) \qquad l' = l_0 (1 + \lambda T')$

$l' - l_0 = l_0 \lambda (T - T')$

□ sphère = infinité de petit Fil



$V_{sphère} \ \frac{4}{3} \pi R^3 \implies v(T) = \frac{4}{3} \pi R_0^3 (1 + \lambda_0 T)^3$

$R = R_0 (1 + \lambda_0 T)$

$(u^n)' = n u' u^{n-1}$

Que vaut $\frac{\Delta v}{// dv}$ pour une variation $\frac{\Delta T}{/ dt}$

$\frac{du}{dT} = 3 u' u^2$

$\frac{dv}{dt} = \frac{4}{\cancel{3}} \pi R_0^3 \times \cancel{3} \times \lambda_0 \times (1 + \lambda_0 T)^2$ ✓

Rappel

$(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$

$(a+b)^7 = a^7 + 7a^6 b + 21 a^5 b^2 + 35 a^4 b^3 + 35 a^3 b^4 + \cancel{16} a^2 b^5 + ab^6 + b^7$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 1
1 7 21 35 35 ~~16~~1

Donc $dv = 4\pi R_0^3 \lambda_0 (1 + \lambda_0 T)^2 dT$

on a le droit de dire ça quand la variat° de volume est petite

Si Δt est petite alors

$\frac{dv}{dT} \simeq \frac{\Delta v}{\Delta T}$

$\Delta v \simeq \Delta t \ (4\pi \ldots)$

↳ Demonstration

$v = \alpha (1 + \lambda_0 T)^3 \quad v' = \alpha (1 + \lambda_0 T')^3 \quad$ ac $\alpha = \frac{4}{3} \pi R_0^3$

$v' - v = \alpha ((1 + \lambda_0 T')^3 - (1 + \lambda_0 T)^3)$

$a^3 - b^3 = ($

laissez tomber

(2) $R = R_0(1 + aT + bT^2) = R_0 + R_0 aT + R_0 bT^2$

$\frac{dR}{dT} = R_0 (a + 2bT)$

sachant les 2 variat° petites

$dR = R_0(a+2bT)\,dT \Leftrightarrow \Delta R = R_0(a+2bT)\Delta T$

Démo $R = R_0(1+aT+bT^2)$

$R' = R_0(1+aT'+bT'^2)$

$R'-R = R_0\left(a(T'-T) + b(T'^2-T^2)\right)$

$= R_0\left(a(T'-T) + b(T'-T)(T'+T)\right)$

$\Delta R = R_0\left(a\,\Delta T + b\,\Delta T\,(T'+T)\right)$

$T'+T = 2T + T' - T$

$= 2T + \Delta T$ — négligeable car petite variat° de température

$\simeq 2T$

$\Delta R = R_0(a + b\times 2T)\Delta T$

(3) $PV = mRT$ $\quad \cancel{P = \frac{mR}{V}T} \quad \cancel{V = \frac{mR}{P}T}$

$T = \frac{PV}{mR}$ $\quad \cancel{\frac{dP}{dT} = \frac{mR}{V}} \quad \cancel{\frac{dV}{dT} = \frac{mR}{P}}$

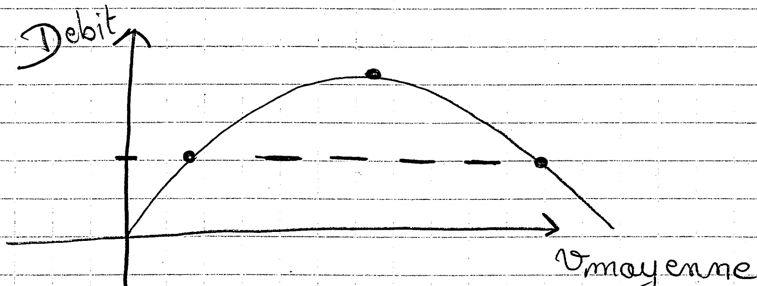
$\frac{\partial T}{\partial P} = \frac{V}{mR}$

$\frac{\partial T}{\partial V} = \frac{P}{mR}$ $\quad \boxed{dT = \frac{\partial T}{\partial V}dV + \frac{\partial T}{\partial P}dP}$

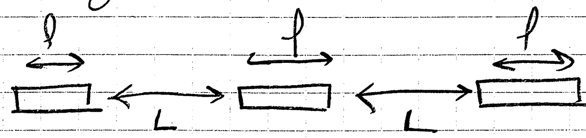
$dT = \frac{1}{mR}(P\,dV + V\,dP)$

$\Delta T = \frac{1}{mR}(P\Delta V + V\Delta P)$

(4)



$v = \frac{d}{t}$



$\frac{m\,s^{-1}}{m}$

$\boxed{L = \frac{v^2}{2a} + vt}$ entre 2 voitures

Débit (en voiture en s) $\quad N = \frac{v}{l+L}$

nb de voiture pd 1 intervalle $\rightarrow \frac{v\times\Delta t}{l+L}$ par unité de temps

⊕ v ↗ ⊕ Debit augmente

⊕ nb de voiture ↗ ⊕ Debit ↘

$$N = \frac{v}{l + \left(\frac{v^2}{2a} + vt\right)}$$

$$\cancel{x^2 + 2at\,x + 2al}$$
$$\cancel{\Delta = (2at)^2 - 4\,2al}$$
$$\cancel{\Delta = 4a^2t^2 - 6al}$$

$v = 0 \rightarrow N = 0$

$v \rightarrow +\infty \quad N \rightarrow 0$

v	$-$	0	$+$	

$$N = \frac{u}{v} = \frac{uv' - u'v}{v^2}$$

$$\frac{dN}{dv} = \frac{l + \frac{v^2}{2a} + vt - v\left(\frac{1}{a}v + t\right)}{\left(l + \frac{v^2}{2a} + vt\right)^2}$$

$$= \frac{l + \frac{v^2}{2a} + \cancel{vt} - \frac{v^2}{a} - \cancel{vt}}{\left(l + \frac{v^2}{2a} + vt\right)^2} = \frac{-\frac{v^2}{2a} + l}{\left(l + \frac{v^2}{2a} + vt\right)^2}$$

$\frac{dN}{dv} = 0 \Leftrightarrow v = \pm\sqrt{2al}$ on enlève le ⊖ car donnée ⊕

v	0		$\sqrt{2al}$		$+\infty$
N'		$+$	0	$-$	
N	0	↗		↘	0

$$N(\sqrt{2al}) = \frac{\sqrt{2al}}{l + \frac{2al}{2a} + \sqrt{2al}\,t}$$

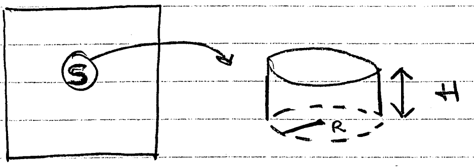
$$N(\sqrt{2al}) = \frac{\sqrt{2al}}{2l + \sqrt{2al}\,t}$$

$$= \frac{\sqrt{2a}}{2\sqrt{l} + \sqrt{2a}\,t}$$

$$= \frac{1}{t + \frac{2\sqrt{l}}{\sqrt{2a}}}$$

$v_{Max} = 10\ m\,s^{-1}$
$= 36\ km/h$

(5)



• surface metal = S
• v maximal

$$S = \pi R^2 + 2\pi R H$$

$$v = \pi R^2 H$$
$$= 2\pi R H \times \frac{R}{2} = (S - \pi R^2)\frac{R}{2}$$

$$v(R) = (2\pi R H) \times \frac{R}{2}$$

$v(R)$ $R \leq ?$

$$S \geqslant \pi R^2$$
$$\frac{S}{\pi} \geqslant R^2$$
$$S - \pi R^2 \geqslant 0$$
$$\Leftrightarrow R \leqslant \sqrt{\frac{S}{\pi}} \qquad \sqrt{\frac{S}{\pi}} \geqslant \sqrt{R^2} = |R| = R$$
$$0 \leqslant R$$

$$v'(a) = \frac{\cancel{2}\pi H R}{\cancel{2}} + \pi R H$$
$$= 2\pi R H$$

NON Scott

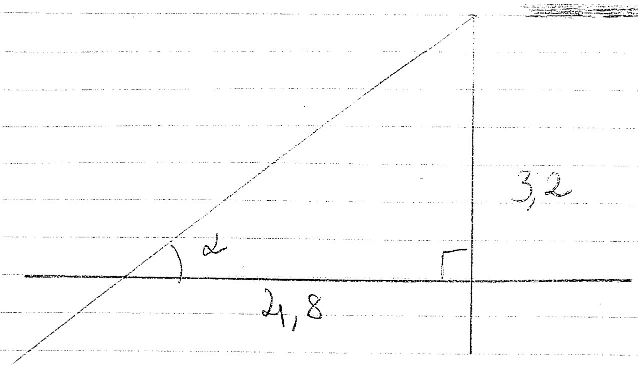
v	0	$\sqrt{\frac{S}{3\pi}}$		$\sqrt{\frac{S}{\pi}}$
$v(R)$	0	$\nearrow$	$\searrow$	0
$v'(R)$	$+$	0	$-$	

$$v'(R) = (S - \pi R^2)\frac{R}{2}$$
$$\frac{dV}{dR} = (-2\pi R)\frac{R}{2} + (S - \pi R^2)\frac{1}{2}$$
$$\frac{dV}{dR} = -\pi R^2 + \frac{1}{2}S - \frac{1}{2}\pi R^2$$
$$\frac{dV}{dR} = -\frac{3}{2}\pi R^2 + \frac{1}{2}S$$

$$\frac{dV}{dR} = 0$$
$$\Leftrightarrow \frac{3}{2}\pi R^2 = \frac{1}{2}S$$
$$\Leftrightarrow R^2 = \frac{S}{3\pi}$$
$$\Leftrightarrow R = \sqrt{\frac{S}{3\pi}}$$

$$V_{max} = \frac{R}{2}\left(S - \cancel{\pi}\frac{S}{3\cancel{\pi}}\right) = \frac{R}{\cancel{2}}\left(\frac{\cancel{2}S}{3}\right) = \boxed{\frac{RS}{3}}$$

$$H = \frac{v}{\pi R^2}$$
$$H = \frac{\cancel{R}S}{3} \times \frac{1}{\pi R^{\cancel{2}}}$$
$$H = \frac{S}{3\pi R}$$
$$= \frac{R^2 \times 3\pi}{3\pi \times R}$$
$$\frac{H}{R} = 1$$

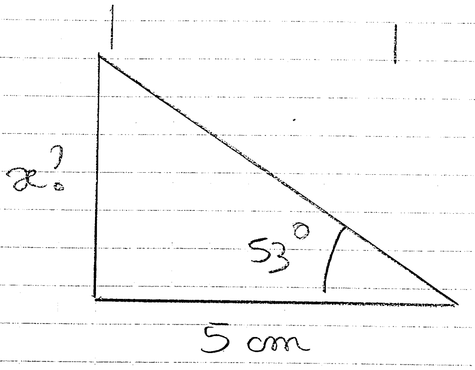


$$\arctan \frac{3,2}{4,8} = 33,69°$$

$$\tan 53 = 1,33 = \frac{\text{opposé}}{\text{adjacent}}$$

$$\cos 53 = 0,60 = \frac{\text{adjacent}}{\text{hyp}}$$

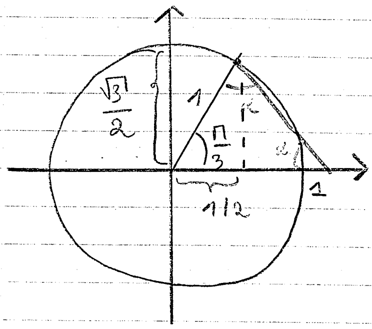
$$\sin 53 = 0,80 = \frac{\text{opposé}}{\text{hyp}}$$



$$1,33 = \frac{x}{5}$$

$$x = 6,65$$

°	0	30	45	60	90
rad	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$



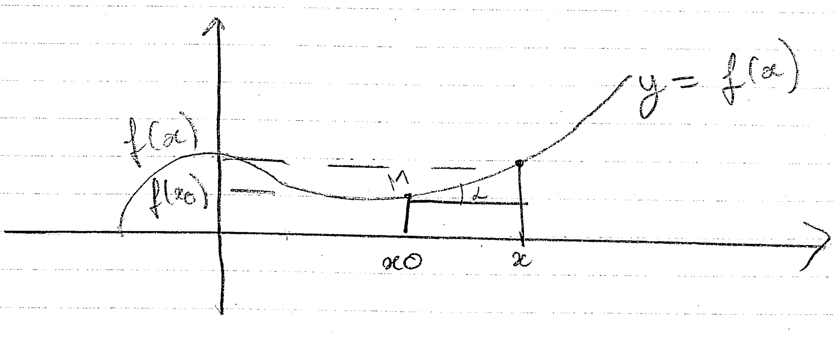
$$2\alpha + \frac{\pi}{3} = \pi$$

$$\alpha = \frac{2\pi}{3 \times 2} = \frac{\pi}{3}$$

$$\sin^2 + \cos^2 = 1$$

$$\sin^2 = 1 - \cos^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\sin = \frac{\sqrt{3}}{2}$$



$$\lim_{\substack{x \to x_0 \\ (x \neq x_0)}} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0)$$

taux d'accroissem^t

$\left(\frac{1}{f}\right)' = -\frac{1}{f^2}$

$(fg)' = f'g + fg'$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$(f^\alpha)' = f' \times \alpha \times f^{\alpha - 1}$

$(f \circ g)' = (f(g))' = (f' \circ g)(g')$
$= (f'(g)\, g')$

$(\sqrt{\ })' = f' / 2\sqrt{f}$

$(\cos)' = -\sin$

$(\sin)' = \cos$

$(\ln)' = 1/x$

$(\exp)' = \exp$

$(\tan u)' = \frac{u'}{\cos^2(u)} = u' \times (1 + \tan^2 u)$

Exercice Derivation

a) $f(x) = \frac{1}{x^2 - 1}$

$f'(x) = -\frac{2x}{(x^2-1)^2}$

$x^2 - 1 = 0$
$x^2 = 1 \quad x = 1 \quad Df = \mathbb{R} \setminus \{ \overset{-1}{1} \}$

b) $f(x) = \tan(2x^2)$

$f(x) = \frac{4x}{\cos^2(2x^2)} = 4x(1 + \tan^2(2x^2))$

c) $f(x) = \ln\left(\frac{1}{\sqrt{1-x^2}}\right) = \sqrt{1-x^2} \times \frac{-2x}{2\sqrt{1-x^2}}$

$\ln u$ ac $u = \frac{1}{\sqrt{1-x^2}}$
$= (1-x^2)^{-1/2}$

$(\ln(u))' = \frac{u'}{u} = -\frac{1}{2} \times \sqrt{1-x^2} \times 2x \times (1-x^2)^{-3/2}$

$= (1-x^2)^{1/2 - \frac{3}{2}} \times \left(-\frac{1}{2}\right) \times -2x$

$= (1-x^2)^{-1} \times x$

e) $f(x) = x^2 (1 + x^{m})^m$
$= 2x(1 + x^{m})^m + x^2 \times m \times (1 + x)^{m-1}$

f est def $\Leftrightarrow 1 - x^2 > 0$
$\Leftrightarrow \sqrt{1} > \sqrt{x^2}$
$\Leftrightarrow 1 > |x|$
$\Leftrightarrow 1 > x > -1$

$Df =]-1, 1[$

ainsi $\frac{1}{\sqrt{1-x^2}}$ est def et > 0

$\forall x \in \mathbb{R}, \ g(x) = e^x - x^2 - x$

$g'(x) = e^x - 2x - 1$

$g''(x) = e^x - 2$

$e^x - 2 = 0$
$e^x = 2$
$x = \ln(2)$

x	$-\infty$	α	$\log(2)$	β	$+\infty$
$g''(x)$	$-$		0	$+$	
$g'(x)$	$+\infty$ $\oplus$	0	$\ominus$ $-0{,}386$	0	$\oplus$ $+\infty$
g	$-\infty$ ↗	0	↘	0	↗ $+\infty$

DIFFERENTIELLE

1
1 1
1 2 1
1 3 3 1

$$f(x) = x^2 + 2x^3$$

$$f(x+h) = (x+h)^2 + 2(x+h)^3 \quad \text{ac } h \text{ petit}$$

$$= x^2 + 2xh + h^2 + 2(x^3 + 3x^2h + 3xh^2 + h^3)$$

$$= x^2 + 2xh + h^2 + 2x^3 + 6x^2h + 6xh^2 + 2h^3$$

$$= f(x) + h \times (2x + 6x^2) + h^2(1 + 6x + 2h)$$

$$h \neq 0 \quad \frac{f(x) - f(x_0)}{x - x_0}$$

$$\frac{f(\underbrace{x+h}_{x_0}) - f(x_0)}{h} = 2x_0 + 6x^2 + h(1 + 6x_0 + 2h)$$

$$h \to 0 \quad x + h \longrightarrow x$$

$$\left.\begin{aligned} \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} &= 2x_0 + 6x_0^2 \\ \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= 2x + 6x_0^2 \end{aligned}\right\} f'(x_0)$$

$$\frac{f(x+h) - f(x)}{h} \xrightarrow{h \to 0} f'(x)$$

d'où $\frac{f(x+h) - f(x)}{h} \simeq f'(x)$

$$f(x+h) - f(x) \sim h\, f'(x)$$

$$\Delta f \sim \Delta x\, f'(x)$$

$$df = d_x f'(x)$$

$$dg \simeq \frac{\partial f}{\partial x_1} \frac{\partial x_1}{h_1} + \frac{\partial F}{\partial x_2} \frac{dx_2}{h_2}$$

$$g(x_1 + h_1, x_2 + h_2) - g(x_1, x_2)$$

DV lim ordre 1 $\quad f(x+h) = f(x) + h f'(x) + \ldots$

DEVELOPPEMENT LIMITÉ

(6) $\left(P + \frac{a}{V^2}\right)(V-b) = RT$

$$P = RT \cancel{/} (V-b) - \frac{a}{V^2}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V-b} \qquad P = \frac{RT}{V-b} - \frac{a}{V^2}$$

$$\left(\frac{\partial P}{\partial V}\right)_T = -\frac{RT}{(V-b)^2} + \frac{2a}{V^3} \qquad V^{-2} = -2V^{-3}$$

~~Forme~~ dif

$$dP = \frac{\partial P}{\partial T} dT + \frac{\partial P}{\partial V} dP$$

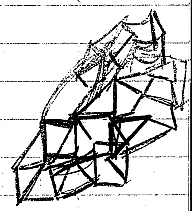
$$P(T+dT, P+dP) = \frac{R}{V-b} dT + \left(-\frac{RT}{(V-b)^2} + \frac{2a}{V^3}\right) dP$$

$(\ P(T,P))$

$$P : \mathbb{R}^2 \to \mathbb{R}$$

$$P(T + \underset{dT}{h_1},\ V + \underset{dV}{h_2}) = \underbrace{dP(P,V) \cdot h}_{\mathcal{L}(\mathbb{R}^2, \mathbb{R})} + o(h)$$

(7) $\rho = \frac{4}{\pi} \frac{m}{hD^2}$



(ex) $U = RI$

$$dU = \frac{\partial U}{\partial R} \frac{dR}{\Delta R} + \frac{\partial U}{\partial I} \frac{dI}{\Delta I}$$

$\| \Delta U$

$$\Delta U \leq \left|\frac{\partial U}{\partial R}\right| \Delta R + \left|\frac{\partial U}{\partial I}\right| \Delta I$$

$$\Delta \rho \leq \left|\frac{\partial \rho}{\partial m}\right| \Delta m + \left|\frac{\partial \rho}{\partial D}\right| \Delta D + \left|\frac{\partial \rho}{\partial h}\right| \Delta h$$

$m, h, D > 0$

$$\left|\frac{\partial \rho}{\partial m}\right| = \frac{4}{\pi h D^2}$$

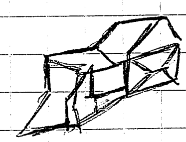
$$\left|\frac{\partial \rho}{\partial D}\right| = \frac{4}{\pi} \frac{m}{h^2 D^2} \qquad \Delta \rho \leq \frac{4}{\pi h D^2} \Delta m + \frac{4m}{\pi h^2 D^2} \Delta h + \frac{8m}{\pi h D^3} \Delta D$$

absolue

$$\left|\frac{\partial \rho}{\partial h}\right| = \frac{8}{\pi} \frac{m}{h D^3}$$

$$\frac{\Delta \rho}{\rho} \leqslant \cancel{\text{moyen}}\ ① \frac{\Delta m}{m} + ① \frac{\Delta h}{h} + ② \frac{\Delta D}{D}$$ relative

$$\frac{\Delta \left(x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}\right)}{x_1^{\alpha_1} \cdots x_n^{\alpha_n}} \leq \sum \frac{\Delta x_i}{x_i} |\alpha_i|$$

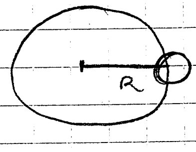


DEVELOPPEMENT LIMITE = approximat° locale

$\boxed{\text{photo 11/09}}$

DL à l'ordre 2

① $g = \dfrac{g_0 R^2}{(R+z)^2}$ $\qquad z \ll R$



$z + R \simeq R$

① $g(z) = g_0 \dfrac{R^2}{(R+z)^2}$ $\qquad g(R+z) \to$ on se place à $z = 0$

faible altitude

$g(z) \simeq g(0) = g_0$

$\left.\dfrac{dg}{dz}\right|_{z=0} = g'(0)$

$g'(z) = g_0 R^2 \times -2 \times (R+z)^{-3}$

$g'(0) = -2 g_0 R^2 R^{-3}$ $\qquad |g'(0)| \leqslant \dfrac{2g_0}{R}$ pr les faibles altitudes

$|\Delta g| \leq |g'(0)|\, \Delta z$ $\qquad \Delta f \leqslant$ Ht les données partielles

② $= \dfrac{2g_0}{R} \Delta z \leqslant 10^{-3}$

$\Delta z \leqslant 10^{-3} \times \dfrac{R}{2g_0}$

$= 10^{-3} \times \dfrac{6{,}4 \times 10^3}{2 \times 10} = \boxed{0{,}32 \text{ km}}$

$\simeq 300$ m

Calculons exactement

$|g(0{,}32) - \underset{10}{\underset{\shortparallel}{g(0)}}| = 0{,}00099 \ll 10^{-3}$

↳ erreur bien ⊕ petite que 10^{-3}