② $\left(P + \frac{a}{V^2}\right)(V-b) = RT$

$$V = b + \frac{RT}{P + \frac{a}{V^2}}$$

$V \longrightarrow V+h \qquad \frac{1}{P + \frac{a}{V^2}} = \frac{1}{P + a(V+h)^{-2}}$

$(V+h)^{-2} = f(h) = f(0) + h f'(0) + \frac{h^2}{2} f''(0) + o(h^2)$

$f(h) = (V+h)^{-2} \qquad f(0) = V^{-2}$

$f'(h) = -2(V+h)^{-3} \qquad f'(0) = -2V^{-3}$

$f''(h) = 6(V+h)^{4} \qquad f''(0) = 6V^{4}$

$$f(h) = f(0) + h f'(0) + \frac{h^2}{2} f''(0) + o\, h^2$$
$$= \underbrace{V^{-2} - 2h V^{-3}}\ \cancel{+ 3h^2 V^4 + o h^2}$$
$$= V^{-2} - \frac{2h}{V^3} + o(h)$$

$$\beta = \frac{1}{P + a\left(V^{-2} - \frac{2h}{V^3} + o(h)\right)} = \frac{1}{P + \frac{a}{V^2} - \frac{2a}{V^3} h + o(h)}$$

$$\beta = \frac{1}{\left(P + \frac{a}{V^2}\right)\left(1 + \alpha h + o(h)\right)} \quad \text{où } \alpha = \left(P + \frac{a}{V^2}\right)^{-1}\left(-\frac{2a}{V^3}\right)$$

$$\frac{1}{1 + \alpha h + o(h)} = 1 - \alpha h + o(h)$$

Trop dur

3) $f(x) = x^5 + 2x^3 + 1$ $\qquad f'(x) = 5x^4 + 6x^2$

1) $f(1+h) = f(1) + h f'(1) + o(h) \simeq f(1) + h f'(1)$

$= 4 + h\,11 + o(h)$

$f(1{,}02) = f(1) + 0{,}02 f'(1) \simeq 4{,}22$

$h = 0{,}02 \qquad 4 + 0{,}02 \times 11$

Soit $F(x) = x^5 + 2x^3 + 1$

Ecrire le DL de la Fct° f au voisinage de $x = b$

$f(b) = f(a) + (b-a) f'(1) + o(b-a) \simeq f(a) + (b-a) f'(1)$

$a = 1 \qquad b = 1{,}02$

$f(b) \simeq 4 + 11(b-a) \qquad f(1{,}02) \simeq 4 + 11 \times 0{,}02$

INTEGRATION

1) $W(\vec{F}) = \cancel{\vec{F} \cdot \vec{AB}}$ on ne peut pas l'utiliser car A et B Fixe

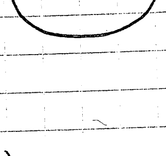
$\int \delta W = \int_0^L F \cdot dl$

$= \int_0^L kx\,dx$

$= \left[\frac{kx^2}{2}\right]_0^L$

$= \frac{kL^2}{2}$

$dV = 4\pi r^2 dr$



$\int dm = \int \rho\, dV$

$= \int_0^R \rho\, 4\pi r^2 dr$

Dim $\quad m^2$

3) $\rho = \rho_0 \left(1 - \alpha \frac{r^2}{R^2}\right)$ m^2

$\int_0^R 4\pi r^2 \rho_0 \left(1 - \alpha \frac{r^2}{R^2}\right) = \rho_0 \int_0^R \left(1 - \alpha \frac{r^2}{R^2}\right) 4\pi r^2 dr \qquad = \rho_0 \int_0^R 4\pi r^2 \left(1 - \alpha \frac{r^2}{R^2}\right) dr$

$= \rho_0 \times 4\pi \int_0^R \left(1 - \alpha \frac{r^2}{R^2}\right) r^2 dr$

$= \rho_0 4\pi \int_0^R r^2 - r^2 \alpha \frac{r^2}{R^2} \, dr$

$= \rho_0 4\pi \left(\frac{R^3}{3} - \frac{R^3}{5}\right)$

Cours Intégration

18/23 sept

- par parche

$$\int_a^b u'v = -\int_a^b uv' + [uv]_a^b$$

IPP

- $\int_1^2 x\ln x \, dx$

- $\int_0^1 x^2 e^x \, dx$

à la main

- $\int_0^{\pi/3} \tan(x) \, dx$

- $\int_1^3 \frac{dx}{2x-1}$ $\quad \frac{u'}{u}$

* $\int_1^2 \ln x \, dx = -\int_1^2 1 \, dx + [x\ln x]_1^2$

$\boxed{x\ln x - x}$ si $x>0$

$u(x) = \ln x \quad v(x) = x$
$u'(x) = \frac{1}{x} \quad v'(x) = 1$

$= 2\ln 2 - [x]_1^2$

$u(x) = x \quad v(x) = \ln x$
$u'(x) = 1 \quad v'(x) = \frac{1}{x}$

$= 2\ln 2 - 2 + 1$

$= 2\ln 2 - 1$ ✓

* $\int_0^1 x^2 e^x \, dx = [x^2 e^x]_0^1 - \int_0^1 2x e^x$

$u(x) = e^x \quad v(x) = x^2$
$u'(x) = e^x \quad v'(x) = 2x$

$= e - [2xe^x]_0^1 + \int_0^1 2e^x$

$= e - 2e + 2e - 2$

$= e - 2$

$\frac{\sin}{\cos} =$

* $\int_0^{\pi/3} \tan(x) \, dx = \int_0^{\pi/3} \frac{\sin x}{\cos x} \, dx$

$u'(x) = \frac{1}{\cos x} \quad v(x) = \sin x$
$u(x) = \ln(\cos x) \quad v'(x) = \cos x$

$= [\ln \cos x \times \sin x]_0^{\pi/3} - \int_0^{\pi/3} \ln(\cos x) \times \cos x$

$\bullet \int_0^1 x^2 e^x dx = -\int_0^1 2x e^x dx + [x^2 e^x]_0^1$ $\qquad u(x) = x^2 \quad u'(x) = 2x$

$v'(x) = e^x \quad v = e^x$

$= -2 \left\{ \int_0^1 x e^x dx \right\} + e$ $\qquad u(x) = x \quad u' = 1$

$v'(x) = e^x \quad v = e^x$

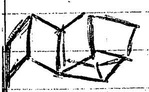
$= -2 \left\{ -\int_0^1 e^x dx + [x e^x]_0^1 \right\} + e$

$= 2(e-1) - 2e + e = e - 2$

$\bullet \int_0^{\pi/3} \frac{\sin x}{\cos x} dx = -\int_0^{\pi/3} \frac{\cos'}{\cos} = -[\ln \cos]_0^{\pi/3}$

$= -\ln \frac{1}{2} + 0$

$= \ln 2 > 0$



$\bullet \frac{1}{2} \int_1^3 \frac{2 dx}{2x-1} = \frac{1}{2} [\ln(2x-1)]_1^3$

$= \frac{1}{2} \ln 5$

pour la prochaine Fois

$\int_0^1 e^{-x} \times x^3 dx$ $\qquad u(x) = x^3 \quad v'(x) = e^{-x} = \frac{1}{e^x}$

$u'(x) = 3x^2 \quad v(x) = -e^{-x}$

$= -\int_0^1 3x^2 \times -e^{-x} + [-x^3 \times e^{-x}]_0^1$

$= 3 \int_0^1 x^2 \times e^{-x} + -1 \times e^{-1}$ $\qquad u(x) = x^2 \quad v'(x) = e^{-x}$

$u'(x) = 2x \quad v(x) = -e^{-x}$

$= 3 \left(-\int_0^1 2x \times -e^{-x} + [x^2 \times -e^{-x}]_0^1 \right) - e^{-1}$

$= 3 \times \int_0^1 2x \times e^{-x} + -1 \times e^{-1} - e^{-1} = 3 \int_0^1 2x \times e^{-x} - 4 e^{-1}$

$u(x) = 2x \quad v'(x) = e^{-x}$

$u'(x) = 2 \quad v(x) = -e^{-x}$

$$= 3\int_0^1 2x - e^{-x} + \left[-2xe^{-x}\right]_0^1 - 4e^{-1}$$

$$= 3\left(2 - 2e^{-1}\right) \quad -10e^{-1}$$

$$= 6 - 6e^{-1} - 10e^{-1} = 6 - 16e^{-1}$$ ✓

$$\int_0^1 x^3 e^{-x}\,dx$$

Primitive de $x^3 e^{-x}$ de la forme $Q(x)e^{-x} = (ax^3 + bx^2 + cx + d)\,e^{-x}$

$$\left(Q(x)\,e^{-x}\right)' = x^3 e^{-x}$$

$$\Leftrightarrow Q'(x)\,e^{-x} - Q(x)\,e^{-x} = x^3 e^{-x}$$

$$\left[-x^3 - 3x^2 - 6x - 6\right]_0^1 = 6 - 16e^{-1}$$

$$Q'(x) - Q(x) = x^3$$

$$3ax^2 + 2bx + c - ax^3 - bx^2 - cx - d = x^3$$

$$\Leftrightarrow \begin{cases} -a = 1 \\ 3a - b = 0 \\ 2b - c = 0 \\ c - d = 0 \end{cases} \Leftrightarrow \begin{cases} a = -1 \\ b = -3 \\ c = -6 \\ d = 6 \end{cases}$$

$$K(x) = (-x^3 - 3x^2 - 6x - 6)\,e^{-x}$$

Intégration par changement de variable

1^{re}

$$\int_a^b f(u)\,du \qquad u = \varphi(x)$$

- $a \longrightarrow \varphi^{-1}(a)$
- $b \longrightarrow \varphi^{-1}(b)$
- $f(u) \rightarrow f(\varphi(x))$
- $du \rightarrow \varphi'(x)\,dx$

$$\int_0^R \sqrt{R^2 - u^2}\,du$$

$$u = \underbrace{R\cos x}_{\varphi(x)}$$

- $u = 0 \qquad R\cos x = 0 \qquad x = \pi/2$
- $u = R \qquad R\cos x = R \qquad x = 0$

⚠ $\neq \cos(x^2)$ et $\cos^2 x$

$$\Rightarrow \sqrt{R^2 - u^2} = \sqrt{R^2 - R^2\cos^2 x}$$

- $\dfrac{du}{dx} = -R\sin x$

$du \longrightarrow -R\sin x\,dx$

$$\Rightarrow \int_{\frac{\pi}{2}}^{0} \sqrt{R^2 - R^2\cos^2 x}\,(-R\sin x)\,dx \qquad -\int_{\frac{\pi}{2}}^{0} = +\int_0^{\pi/2}$$

$$= R^2 \int_0^{\pi/2} \sin x \sqrt{1 - \cos^2 x}\,dx$$

$$\sqrt{1-\cos^2 x} = \sqrt{\sin^2 x} = |\sin x| = \sin x$$

- $\cos(a+b) = \cos a\cos b - \sin a\sin b$
- $\sin(a+b) = \sin a\cos b + \sin b\cos a$
- $\sin 2x = 2\sin x\cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 1 - \sin^2 x - \sin^2 x$
  $= 1 - 2\sin^2 x$

$$= R^2 \int_0^{\pi/2} \sin^2 x\,dx$$

$$= \frac{R^2}{2}\left[x - \frac{\sin(2x)}{2}\right]_0^{\pi/2} \xleftarrow{\text{intégré}} \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$= \frac{\pi R^2}{4} > 0$$

5

2e

$$\frac{dx}{du} = \varphi'(u)$$

$$du = \frac{dx}{\varphi'(u)} = \frac{dx}{\varphi'(\varphi^{-1}(x))}$$

$$\int_a^b f(u)\,du = \int_{\varphi a}^{\varphi b} f(\varphi^{-1}(x)) \frac{dx}{\varphi'(\varphi^{-1}(x))}$$

Ex

$$I = \int_0^{\pi/2} \underbrace{\cos^2 u}_{f(g(x))} \underbrace{\sin u}_{g'(x)} \underbrace{du}_{dx}$$

$x = \cos u$

On a donc $dx = d(\cos u) = -\sin u\,du$ on remplace donc $\sin u\,du$ par $-dx$

On remplace $\cos^2 u$ par x^2

si $u = 0$ $x = \cos 0 = 1$ et si $u = \pi/2$ on a $x = \cos\frac{\pi}{2} = 0$

$$\int_0^{\pi/2} \cos^2 u \sin u\,du = \int_1^0 x^2(-dx) = \int_0^1 \underbrace{x^2 dx}_{} = \underbrace{\left[\frac{x^3}{3}\right]_0^1}_{F(g(x))} = \frac{1}{3}$$

Exercice

$$\frac{t^2}{2} - \frac{1}{2} = 1 \quad \frac{t^2}{2} = 1 + \frac{1}{2}$$

$$t^2 = \frac{3}{\cancel{2}} \times \cancel{2}$$

$$t = \sqrt{3}$$

(2) $$\int_1^4 \frac{x}{\sqrt{2x+1}}\,dx \qquad 1 + 2x = t^2 \qquad x = \frac{t^2-1}{2}$$

$$\sqrt{t^2} = |t| = t^+ \quad \in [\sqrt{3}, 3]$$

$$dx = d\left(\frac{t^2-1}{2}\right) = t\,dt$$

$$= \int_{\sqrt{3}}^{\sqrt{9}=3} \frac{\frac{t^2-1}{2}}{\cancel{t}} \cancel{t}\,dt$$

$$= \left[\frac{t^3}{6} - \frac{1}{2}t\right]_{\sqrt{3}}^{\cancel{\sqrt{5}}\,3} = \cancel{\frac{5\sqrt{5}}{6} - \frac{1}{2}\sqrt{5} - \frac{3\sqrt{3}}{6} + \frac{1}{2}\sqrt{3}}$$

$$= \frac{27}{6} - \frac{3}{2} - \frac{\sqrt{3}\,3}{6} + \frac{1}{2}\sqrt{3} = 3$$

$$\int_0^{\pi/2} \frac{\cos x \, dx}{6-5\sin x+\sin^2 x} \quad \sin(x) = t$$

$\cos x \, dx = dt$

$$\int_0^{1} \frac{dt}{6-5t+t^2} = \int_0^1 \frac{1}{6-5t+t^2}\,dt$$

$\Delta = 25-4\times 6 = 1$

$\frac{5-\sqrt{1}}{2} = \frac{4}{2} = 2$ $\quad \frac{5+\sqrt{1}}{2} = \frac{6}{2} = 3$

$$= \int_0^1 \frac{1}{(t-2)} \times \frac{1}{(t-3)}$$

On cherche α et β tel que

$$\frac{1}{(t-2)}\,\frac{1}{(t-3)} = \frac{\alpha}{t-2} + \frac{\beta}{t-3}$$

$$= \frac{\alpha(t-3)+\beta(t-2)}{(t-2)(t-3)}$$

$\alpha(t-3)+\beta(t-2) = 1$ avec $\alpha = -\beta$ decompo d'elem^t simple

$\alpha(t-3) - \alpha(t-2) = 1$

$-3\alpha + 2\alpha = 1$

$\alpha = -1$ alors $\frac{1}{6-5t+t^2} = \frac{-1}{(t-2)} + \frac{1}{(t-3)}$

$$\int_0^1 \frac{1}{6-5t+t^2} = \Big[\ln|t-3| - \ln|t-2|\Big]_0^1$$

$$= \ln(2) - \ln(3) + \ln(2)$$

$$= 2\ln(2) - \ln(3) = \ln 4/3$$

$$\frac{1 \times \cancel{(t-3)} \times (t-2)}{\cancel{(t-3)}(t-2)} = \frac{a \times (t-3) \times (t-2)}{(t-3)} \quad \frac{b \times (t-3)}{(t-2)} \times (t-2) = 0$$

$a = \frac{1}{3-2} = 1 \qquad b = \frac{1}{2-3} = -1$

TD Exercice 4

$$\frac{dN}{dt} = -\lambda N \quad \frac{[N]}{[T]} = [\lambda][N] \qquad N(t=0) = N_0$$

$$\frac{dN}{dt} + \lambda N = 0$$

Dim λ = s^{-1} = T^{-1}

$N(t) = N_0 e^{-\lambda t}$

t_0 $\frac{dN}{dt} = 510 = \lambda N$

t_1 $\frac{dN}{dt} = 560 = \lambda N$

$\longrightarrow t_{1/2}$: $N(t) = N_0/2$

$$\cancel{N_0} e^{-\lambda t_{1/2}} = \frac{\cancel{N_0}}{2}$$

$$e^{-\lambda t_{1/2}} = \frac{1}{2}$$

$$t_{1/2} = -\frac{1}{\lambda} \ln\left(\frac{1}{2}\right)$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

$N_0 = 816 \quad N_t = 560 \quad t_{1/2} = 5570$ ans $= 5570 \times 31\,536\,000$ s $= 1,756 \times 10^{11}$ s

On cherche t ?

Or $t_{1/2} = \frac{\ln 2}{\lambda}$ $\quad \lambda = \frac{\ln 2}{t_{1/2}} = 3,9473 \times 10^{-12}$ s $= 1,24 \times 10^{-4}$ an

On a $N(t) = N_0 e^{-\lambda t}$ $\quad \frac{N(t)}{N_0} = e^{-\lambda t}$

$$\frac{\ln\left(\frac{N(t)}{N_0}\right)}{-\lambda} = t$$

$= 9,5376 \times 10^{10}$ s

$= 3036,1094$ an

$$t = -\frac{1}{\lambda} \ln \frac{N(t)}{N_0}$$

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N(t)} = t_{1/2} \times (\ln 2)^{-1} \times \ln \frac{N_0}{N_t}$$

(2) $\displaystyle\int_1^5 \frac{\sqrt{x-1}}{x}\,dx$ avec $x-1 = t^2$

$x = t^2+1$

$dx = 2t\,dt$

$$= \int_0^2 \frac{t}{t^2+1}\,2t\,dt$$

$$= \int_0^2 \frac{2t^2}{t^2+1}\,dt = 2\int_0^2 \frac{t^2+1-1}{t^2+1} = 2\int_0^2 \frac{t^2+1}{t^2+1}$$

$$= \cancel{\int_0^2 \frac{2}{1+\frac{1}{t^2}}\,dt} = \qquad -2\int_0^2 \frac{1}{t^2+1}$$

$$= 2\int_0^2 1 - 2\int_0^2 \frac{1}{t^2+1}$$

$$= 2\left[t\right]_0^2 - 2\left[\arctan(t)\right]_0^2$$

$$= 4 - 2\arctan(2)$$

(3) $\displaystyle\int_1^{4/3} \frac{1}{x\sqrt{-1+x^2}}$ avec $x = \frac{1}{t}$ $\qquad t = \frac{1}{\frac{4}{3}} = \frac{3}{4}$

$dx = -\frac{1}{t^2}\,dt$

$$= \frac{1}{\frac{1}{t}\times\sqrt{-1+\frac{1}{t^2}}} \times -\frac{1}{t^2}\,dt$$

$$= -\frac{1}{t\times\sqrt{-1+\frac{1}{t^2}}} = -\int_{\boxed{1}}^{\boxed{3/4}} \frac{1}{\sqrt{1-t^2}}$$

$$= -\left[\arcsin(t)\right]_1^{3/4}$$

$$= -\arcsin\left(\frac{3}{4}\right) + \arcsin(1)$$

① $f(x) = \dfrac{\ln(x)}{x}$ $\qquad \lim\limits_{x \to +\infty} F(x) = \dfrac{\quad}{+\infty}$

$f'(x) = \dfrac{\frac{1}{\cancel{x}} \times \cancel{x} - \ln(x)}{x^2}$ $\qquad \lim\limits_{x \to \infty} f(x) = +\infty$

$= \dfrac{1 - \ln(x)}{x^2}$ $\qquad 1 - \ln(x) = 0$

$\ln(x) = 1$

$x = e^1$

	e^1	
$1 - \ln(x)$	$+$ ϕ	$-$
x^2	$+$	

$+ \quad \phi \quad -$

$-\infty \nearrow \qquad \searrow \infty$

$9^9 = 9^p$

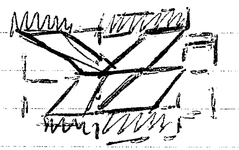
$p = 2 \qquad 9 = 4$

②

$V_{or} + V_{ag} = \qquad V = \dfrac{T}{\rho_{ag}\, g}$

$\theta_{or} + \theta_{ag} = \theta$

$\rho_{or} V_{or} + \rho_{ag} V_{ag} = \theta$



$390 - 19{,}5\, V_{ag} + 10{,}5\, V_{ag} = 300$

$390 - 9\cancel{,5}\, V_{ag} = 300$

$V_{ag} = +\dfrac{90}{9\cancel{,5}} = +10$

$V_{or} = 10$