Exercice 1:

Q1)

$$\begin{array}{ll} 20 & 1{,}01 \\ & \quad > -0{,}027 \\ 21 & 0{,}983 \end{array}$$

$p_1(T) = 1{,}01 - 0{,}027(T - 20)$

$= 1{,}01 + 0{,}54 - 0{,}027\,T$

$p_1(T) = 1{,}55 - 0{,}027\,T$

$\vartheta = 1 \Rightarrow 1 = 1{,}55 - 0{,}027\,T$

$0 = 0{,}55 - 0{,}027\,T$

$0{,}027\,T = 0{,}55 \Rightarrow \boxed{T = 20{,}37\ °C}$

Q2)

$$\begin{array}{llll} 23 & 0{,}938 & & \\ & & -0{,}021 & \\ 24 & 0{,}917 & & -0{,}001 \\ & & -0{,}023 & \\ 25 & 0{,}894 & & \end{array}$$

$p_2(T) = 0{,}938 - 0{,}021(T - 23) - 0{,}001(T-23)(T-24)$

$p_2(T) = 0{,}938 - 0{,}021\,T + 0{,}483 - 0{,}001(T^2 - 47T + 552)$

$p_2(T) = 0{,}863 + 0{,}026T - 0{,}001\,T^2$

$T = 24{,}5°C \Rightarrow \boxed{\vartheta = 0{,}8997\ m^2/s.}$

Exercice 2 $f(x) = x^3 - 8$

BISSECTION: $f(0) = -8$ $f(3) = 19$. $f\left(\frac{3}{2}\right) = -4{,}625 < 0$

$a_1 = \frac{3}{2}$ $b_1 = 3.$ $c_1 = 2{,}25$ $f(c_1) = 3{,}39 > 0.$

$a_2 = \frac{3}{2}$ $b_2 = 2{,}25.$ $c_2 = 1{,}875$ $f(c_2) = -1{,}48 < 0$

$a_3 = 1{,}875$ $b_3 = 2{,}25$ $c_3 = 2{,}0625$ $f(c_3) = 0{,}774 > 0$

$a_3 = 1{,}875$ $b_4 = 2{,}0625.$

NEWTON : $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ avec $f(x) = x^3 - 8.$

$f'(x) = 3x^2 -$

$$x_{n+1} = x_n - \frac{x_n^3 - 8}{3x_n^2} = \frac{2x_n}{3} + \frac{8}{3x_n^2}$$

$x_0 = 3.$

$x_1 = \frac{2x_0}{3} + \frac{8}{3x_0^2} = 2 + \frac{8}{27} \simeq 2{,}2963.$

$x_2 = \frac{2x_1}{3} + \frac{8}{3x_1^2} = 2{,}0366.$

$x_3 = \frac{2x_2}{3} + \frac{8}{3x_2^2} = 2{,}0007.$

$x_4 = 2{,}0000.$

SECANTE : $x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$

$x_0 = 0$

$x_1 = 3.$

$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 3 - \frac{19(3-0)}{19+8} = 0{,}8889.$

$f(x_2) = -7{,}2977$

$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 0{,}8889 - \frac{-7{,}2977(-2{,}111)}{-7{,}2977 - 19}$

$= 1{,}4747$ $\quad f(x_3) = -4{,}7927$

$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)} = 1{,}4747 - \frac{-4{,}7927 \times 0{,}5858}{2{,}5050} = 2{,}5956$

Exercice ①
$$\begin{cases} y'(t) = \underbrace{1 + t - y(t)}_{f(t,y)} \\ y(0) = 1 \end{cases}$$

① EULER PROGRESSIF : $h = 0,6$ $\quad t_0 = 0 \quad y_0 = 1$.

- $t_1 = 0,6 \qquad y_1 = y_0 + h f(t_0, y_0)$
$\qquad y_1 = 1 + 0,6 \times (1 + 0 - 1) = 1$.

- $t_2 = 1,2 \qquad y_2 = y_1 + h f(t_1, y_1)$
$\qquad y_2 = 1 + 0,6 \times (1 + 0,6 - 1) = 1,36$.

EULER MODIFIEE : $\hat{y} = y_n + h f(t_n, y_n)$
$t_{n+1} = t_n + h$
$y_{n+1} = y_n + \frac{h}{2} \left(f(t_n, y_n) + f(t_{n+1}, \hat{y}) \right)$.

- $\hat{y} = y_0 + h f(t_0, y_0) = 1$.
$t_1 = 0,6$
$y_1 = y_0 + \frac{h}{2} \left(f(t_0, y_0) + f(t_1, \hat{y}) \right)$
$= 1 + 0,3 \left(0 + (1 + 0,6 - 1) \right) = 1 + 0,3 \times 0,6 = 1,18$.

- $\hat{y} = y_1 + h f(t_1, y_1) = 1,18 + 0,6 \times (1 + 0,6 - 1,18) = 1,4320$.
$t_2 = 1,2$
$y_2 = y_1 + \frac{h}{2} \left(f(t_1, y_1) + f(t_2, \hat{y}) \right) = 1,18 + 0,3 \times \left(1,6 - 1,18 + (1 + 1,2 - 1,4320) \right) = 1,5364$

② Euler progressif : ordre 1.
Euler modifiée : ordre 2.

③ $y(t_{n+1}) = y(t_n) + h y'(t_n) + \frac{h^2}{2} y''(t_n) + o(h^3)$. $\qquad f(t,y) = 1 + t - y$

avec $y'(t_n) = f(t_n, y_n)$ et $y''(t_n) = \frac{d}{dt} f(t_n, y_n)$

$$= \underbrace{\frac{\partial f}{\partial y}}_{-1} \underbrace{\frac{\partial y}{\partial t}}_{y'(t)} + \underbrace{\frac{\partial f}{\partial t}}_{1} = 1 - y'(t)$$
$$= 1 - f(t_n, y_n)$$

donc $y(t_{n+1}) = y(t_n) + h f(t_n, y_n) + \frac{h^2}{2}\left(1 - f(t_n, y_n)\right) + o(h^3)$

$$y(t_{n+1}) = y(t_n) + ha + \frac{h^2}{2} b \quad \text{avec} \quad a = f(t_n, y_n)$$
$$b = 1 - f(t_n, y_n).$$

$$\Rightarrow a = 1 + t - y.$$
$$b = 1 - (1 + t - y) = y - t.$$

(4) • $t_1 = 0{,}6$.

$$y_1 = y_0 + ah + \frac{h^2}{2} b \quad \text{avec} \quad a = 1 + t_0 - y_0 = 1 - 1 = 0.$$
$$b = y_0 - t_0 = 1$$

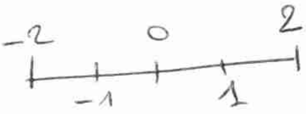
$$y_1 = 1 - 0 \times 0{,}6 + \frac{0{,}6^2}{2} \times 1 = 1{,}18$$

• $t_2 = 1{,}2$.

$$y_2 = y_1 + ah + \frac{h^2}{2} b \quad \text{avec} \quad a = 1 + t_1 - y_1 = 1 + 0{,}6 - 1{,}18 = 0{,}42$$
$$b = y_1 - t_1 = 1{,}18 - 0{,}6 = 0{,}58$$

$$y_2 = 1{,}18 + 0{,}42 \times 0{,}6 + \frac{0{,}6^2}{2} \times 0{,}58 = 1{,}5364.$$

Exercice (4). $I = \int_{-2}^{2} \frac{1}{1+t^2}\, dt.$



SIMPSON COMPOSITE sur 2 sous intervalles $m = 2$.

Q1) SIMPSON COMPOSITE sur 2 sous intervalles

$$I_{2,2} = \frac{h}{6}\left[f(x_0) + 2\sum_{k=1}^{m-1} f(x_k) + 4\sum_{k=0}^{m-1} f(T_k) + f(x_m)\right]$$

$$I_{2,2} = \frac{2}{6}\Big(\underbrace{f(-2)}_{\frac{1}{5}} + 2\underbrace{f(0)}_{1} + 4\big(\underbrace{f(-1)}_{\frac{1}{2}} + \underbrace{f(1)}_{\frac{1}{2}}\big) + \underbrace{f(2)}_{\frac{1}{5}}\Big)$$

$$I_{2,2} = \frac{2}{6}\left(\frac{1}{5} + 2 + 4 + \frac{1}{5}\right) = \frac{1}{3}\left(6 + \frac{2}{5}\right) = \frac{32}{15} = 2{,}1333.$$

Q2) Changement de variable: $I = \int_{-2}^{2} \frac{1}{1+t^2}\, dt$

On pose $T = \frac{t}{2} \Rightarrow I = \int_{-1}^{1} \frac{1}{1+(2T)^2}\, 2\, dT$

$$I = \int_{-1}^{1} \frac{2}{1+4T^2}\, dT$$

On applique la quadrature de Radau :

$$\int_{-1}^{1} f(t)\,dt \simeq \frac{1}{2} f(-1) + \frac{3}{2} f\left(\frac{1}{3}\right) \quad \text{avec } f(t) = \frac{2}{1+4T^2}$$

$$\Rightarrow f(-1) = \frac{2}{5}$$

$$\Rightarrow f\left(\frac{1}{3}\right) = \frac{2}{1+4\cdot\frac{1}{9}} = \frac{2}{\frac{13}{9}} = \frac{18}{13}.$$

$$\Rightarrow I \simeq \frac{1}{2} \times \frac{2}{5} + \frac{3}{2} \times \frac{18}{13} = \frac{1}{5} + \frac{27}{13} = \frac{148}{65} = 2{,}2769$$

Solution exacte : $2\arctan(2) = 2{,}2143$.

Primitive de $\frac{1}{1+t^2}$: $\arctan(t)$.

Exercice 5 :

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -8 \\ 2 & -2 & 3 & -3 & -20 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 4 \end{array}\right) \quad l_1 \leftrightarrow l_2 \quad \left(\begin{array}{cccc|c} \boxed{2} & -2 & 3 & -3 & -20 \\ 1 & -1 & 2 & -1 & -8 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 4 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 0 & 1/2 & 1/2 & 2 \\ 0 & 2 & -1/2 & 3/2 & 8 \\ 0 & 0 & 5/2 & 9/2 & 14 \end{array}\right) \quad \begin{array}{l} l_2 \leftarrow l_2 - \frac{l_1}{2} \\ l_3 \leftarrow l_3 - \frac{l_1}{2} \\ l_4 \leftarrow l_4 - \frac{l_1}{2} \end{array} \quad l_2 \leftrightarrow l_3 \quad \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & \boxed{2} & -1/2 & 3/2 & 8 \\ 0 & 0 & 1/2 & 1/2 & 2 \\ 0 & 0 & 5/2 & 9/2 & 14 \end{array}\right)$$

$$l_3 \leftrightarrow l_4 \quad \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 2 & -1/2 & 3/2 & 8 \\ 0 & 0 & \boxed{5/2} & 9/2 & 14 \\ 0 & 0 & 1/2 & 1/2 & 2 \end{array}\right) \quad l_4 \leftarrow l_4 - \frac{l_3 \times 1/2}{5/2} = l_4 - \frac{l_3}{5} \quad \left(\begin{array}{cccc|c} 2 & -2 & 3 & -3 & -20 \\ 0 & 2 & -1/2 & 3/2 & 8 \\ 0 & 0 & 5/2 & 9/2 & 14 \\ 0 & 0 & 0 & -\frac{2}{5} & -\frac{4}{5} \end{array}\right)$$

$$-\frac{2}{5}x_4 = -\frac{4}{5} \Rightarrow x_4 = 2.$$

$$\frac{5}{2}x_3 + 9 = 14 \Rightarrow x_3 = 5 \times \frac{2}{5} = 2$$

$$2x_2 - 1 + 3 = 8 \Rightarrow x_2 = 3$$

$$2x_1 - \cancel{6} + \cancel{6} - 6 = -20 \Rightarrow x_1 = -7$$

$$x = \begin{pmatrix} -7 \\ 3 \\ 2 \\ 2 \end{pmatrix}$$